

sing Sun: 1.1

sing Tue: 1.2/3, 1.6(pt 1)

dterm 1 is next Thursday!

Entry Tasks:

1. Solve $\frac{10}{x-1} = 5$.

2. Solve for t in the equation

$$S = P + At.$$

3. Simplify $0.5(x - 4) < 0.1x + 3$

1

$$\frac{10}{x-1} = 5$$

$\cdot (x-1)$

CLEAR DENOMINATOR

$$(x-1) \cdot \frac{10}{x-1} = (x-1) \cdot 5$$

$$10 = 5x - 5$$

$\rightarrow +5$

$$15 = 5x$$

$\rightarrow \div 5$

$$\frac{15}{5} = x$$

$$\boxed{x = 3}$$

CHECK: $\frac{10}{3-1} = \frac{10}{2} = 5$ ✓

2

$$S = P + At$$

$\rightarrow -P$

$$S - P = At$$

$\rightarrow \div A$

$$\frac{S - P}{A} = t$$

$$\boxed{t = \frac{S - P}{A}}$$

CHECK: $P + A\left(\frac{S - P}{A}\right) = P + S - P = S$ ✓

3

$$0.5(x - 4) < 0.1x + 3$$

$$0.5x - 2 < 0.1x + 3$$

$\rightarrow +2$

$$0.5x < 0.1x + 5$$

$\rightarrow -0.1x$

$$0.4x < 5$$

$$x < \frac{5}{0.4} = 12.5$$

$$\boxed{x < 12.5}$$

$$x = 12$$

$$0.5(12 - 4) = 4$$

$$0.1(12) + 3 = 4.2$$

$$4 < 4.2$$

CHECK:

TRY SOME NUMBER SMALLER THAN 12.5, DOES IT WORK IN ORIGINAL INEQUALITY?

Inequalities

We simplify inequalities the same way we solve equations, except

if you multiply or divide by a negative then you flip the inequality.

Example:

$$\text{Simplify } -2x + 4 > 10$$

$$\begin{array}{l} -2x > 6 \\ x < \frac{6}{-2} \end{array} \quad \text{FLIP}$$

$$\boxed{x < -3}$$

$$\text{CHECK: } \underbrace{x = -4}_{\substack{\text{RANDOM} \\ \text{NUMBER} \\ \text{SMALLER THAN} \\ -3}} \Rightarrow -2(-4) + 4 = 12 > 10 \quad \checkmark$$

Example

$$\begin{array}{l} 1 < 3 \\ \downarrow \quad \downarrow \\ -2 > -6 \end{array}$$

if you multiply both sides by -2 you get

FLIP

1,2/1.3 Linear Functions & Problems

Problem solving strategies

1. Read carefully:
What's given? What's wanted?
What are the units?
2. Label everything! Introduce variables for unknown values.
3. Translate to algebra/notation.
4. Solve for anything you can.
Even if you can't immediately answer the question, solve for something related.
5. Make sure your final answer actually answers the original question. And check your work!

If stuck, make up numbers and work out an example!

Applied Problems from lecture pack:

1. You have \$25,000 to invest in two different funds. One investment fund has an annual yield of 8% and the other 11%.

How much must you invest in each to earn exactly \$2500 in one year?

(Just like HW 1.1/Problem 10)

(Note: you must invest all the money)

STEP 1/2 Let x = "AMOUNT INVESTED IN 8% ACCOUNT"
Let y = "AMOUNT INVESTED IN 11% ACCOUNT"

STEP 3 TOTAL MONEY INVESTED = $x + y = 25,000$ ①
TOTAL INTEREST = $0.08x + 0.11y = 2,500$ ②

STEP 4 COMBINE FACTS!

$$\textcircled{1} \quad y = 25000 - x \quad \longrightarrow \uparrow$$

$$\textcircled{1} \ \& \ \textcircled{2} \quad 0.08x + 0.11(25000 - x) = 2500$$

$$0.08x + 2750 - 0.11x = 2500$$

$$2750 - 0.03x = 2500 \quad \left. \vphantom{2750 - 0.03x = 2500} \right\} -2750$$

$$-0.03x = -250$$

$$x = \frac{-250}{-0.03} = 8333.\bar{3}$$

STEP 5

$$x = \$8,333.\bar{3} \quad \leftarrow \text{IN 8\% ACCOUNT}$$

$$y = 25000 - x \\ = \$16,666.\bar{67} \quad \leftarrow \text{IN 11\% ACCOUNT}$$

CHECK !!!

$$x + y = 25,000 \quad \checkmark$$

$$0.08x + 0.11y = 2,500 \quad \checkmark$$

2(a). You are told you have to average above 75% on the three exams to pass the class. Assume you get 65% on your first test and 78% on your second test. What does your final exam score need to be in order to pass the class?

(Similar to HW 1.1/9)

STEP 1/2

Let x = "FINAL EXAM PERCENTAGE"

STEP 3

WANT AVERAGE OF THREE TESTS $> 75\%$

$$\Rightarrow \frac{65 + 78 + x}{3} > 75 \quad \cdot 3$$

STEP 4

$$\underbrace{65 + 78 + x}_{143 + x} > 225 \quad \cdot -143$$

$$\boxed{x > 82}$$

STEP 5

NEED TO SCORE ABOVE 82% ON FINAL!

2(b) To earn a grade of 3.0 or above in a course, a student must obtain at least an 85% average on two midterms and a final exam. The final exam is worth twice as much as the midterms. The student earns a 78 and a 93 on the midterms. How high must the student's final exam grade be to ensure a 3.0 in the course?

(Similar to HW 1.1/9)

STEP 1/2

Let x = "FINAL EXAM PERCENTAGE" COUNTS AS TWO EXAMS!! \hookrightarrow

STEP 3

WANT AVERAGE OF "FOUR" TESTS ≥ 85

$$\Rightarrow \frac{78 + 93 + x + x}{4} \geq 85 \quad \cdot 4$$

STEP 4

$$\underbrace{78 + 93}_{171} + 2x \geq 340 \quad \cdot -171$$

$$2x \geq 169 \quad \cdot \div 2$$

$$x \geq \frac{169}{2}$$

STEP 5

NEED TO SCORE ABOVE 84.5% ON FINAL!

$$\boxed{x \geq 84.5}$$

3. You sell Things. Your total cost, in hundreds of dollars, for producing x hundred Things is given by the formula

$$C(x) = 40 + 0.5x.$$

How much do you pay in FC?

What is your TC if you produce 700 Things?

How many Things are you producing if your total costs are \$12,000?

$$\begin{aligned} x &= \underline{\hspace{2cm}} \text{ hundred Things} \\ C(x) &= \underline{\hspace{2cm}} \text{ hundred dollars} \end{aligned}$$

1 $C(x) = 40 + 0.5x$

$$TC(0) = 40 + 0.5(0) = 40 \text{ hundred dollars} = \$4,000 = FC$$

2 700 THINGS $\Leftrightarrow x = \underline{7}$ hundred things

$$x = 7 \Rightarrow C(7) = 40 + 0.5(7) = 43.5 \text{ hundred dollars} = \$4,350$$

3 \$12,000 = 120 hundred dollars

$$\Rightarrow C(x) = 40 + 0.5x \stackrel{?}{=} 120 \quad \left. \begin{array}{l} \\ \end{array} \right\} -40$$

$$\Rightarrow 0.5x = 80$$

$$x = \frac{80}{0.5} = 160 \text{ hundred Things} = 16,000 \text{ THINGS}$$

Linear functions are written as:

$$f(x) = mx + b \quad (\text{slope-intercept})$$

or

$$f(x) = m(x - x_0) + y_0 \quad (\text{point-slope})$$

How to find the equation of a line

Given any two points $(x_1, y_1), (x_2, y_2)$:

- Compute slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- Write: $y = m(x - x_1) + y_1$

Examples:

1. Find the equation of the line that goes through $(1,1)$ and $(5,7)$.

$$\text{SLOPE} = \frac{7-1}{5-1} = \frac{6}{4} = \frac{3}{2} = 1.5$$

$$\boxed{y = 1.5(x - 1) + 1}$$

or

$$y = 1.5x - 1.5 + 1 = 1.5x - 0.5$$

$$\boxed{y = 1.5x - 0.5}$$

CHECK!

DOES $(1,1)$ work ✓

DOES $(5,7)$ work ✓

2. Find the equation of the line that goes through $(4,1)$ and $(6,15)$.

$$\text{SLOPE} = \frac{15-1}{6-4} = \frac{14}{2} = 7$$

$$\boxed{y = 7(x - 4) + 1} = 7x - 28 + 1 = 7x - 27$$

or

$$\boxed{y = 7x - 27}$$

CHECK!

DOES $(4,1)$ work ✓

DOES $(6,15)$ work ✓

Line Facts:

— Anything that can be written as
 $ax + by = c$
gives a line.

— Two lines are parallel if they have
the same slope.

— $y = h$ gives a horizontal line.

— $x = k$ gives a vertical line.

} Example

$$\boxed{5x + 3y = 14}$$

$$\Rightarrow 3y = 14 - 5x$$

$$\Rightarrow \boxed{y = \frac{14}{3} - \frac{5}{3}x}$$

↑
Slope = $-\frac{5}{3}$

←
SAME

4. You sell Objects. Each object sells for \$5. Your total cost function is linear. When you produce 1 Object, total cost is \$53. When you produce 10 Objects, your total cost is \$80.

- a Find formulas for: TR, TC, Profit
- b What are: FC, MR, MC, MP?
- c For what levels of production (values of x) do you make a profit?

a "total cost is linear"

$$\Rightarrow TC(x) = m(x - x_1) + y_1$$

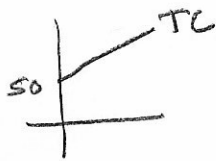
2 PTS: (1, 53) (10, 80)

$$\text{SLOPE} = \frac{80 - 53}{10 - 1} = \frac{27}{9} = 3$$

$$TC(x) = 3(x - 1) + 53$$

$$TC(x) = 3x - 3 + 53$$

$$TC(x) = 3x + 50$$



price = \$5/item

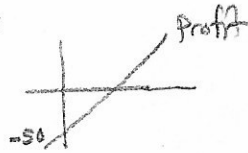
$$\Rightarrow TR(x) = 5x$$



$$\begin{aligned} \text{Profit} = P(x) &= TR(x) - TC(x) \\ &= (5x) - (3x + 50) \\ &= 5x - 3x - 50 \end{aligned}$$

Don't Forget!!!

$$\Rightarrow P(x) = 2x - 50$$



$$b) FC = TC(0) = 3(0) + 50 = \$50$$

$$\begin{aligned} MR(x) &= TR(x+1) - TR(x) \\ &= 5(x+1) - 5x = 5x + 5 - 5x \end{aligned}$$

$$\Rightarrow MR(x) = 5 \quad \leftarrow \text{ADDITIONAL REVENUE WHEN WE SELL 1 ITEM}$$

$$\begin{aligned} MC(x) &= TC(x+1) - TC(x) \\ &= (3(x+1) + 50) - (3x + 50) \\ &= 3x + 3 + 50 - 3x - 50 \end{aligned}$$

$$MC(x) = 3 \quad \leftarrow \text{ADDITIONAL COST WHEN WE PRODUCE 1 ITEM}$$

$$MP(x) = MR(x) - MC(x) = 5 - 3 = \$2/\text{item}$$

ADDITIONAL PROFIT

c) PROFIT > 0

$$2x - 50 > 0$$

$$2x > 50$$

$$x > 25$$

MUST SELL MORE THAN 25 TO MAKE A POSITIVE PROFIT

